Mathematical and Computer Modelling 58 (2013) 1714–1726

Contents lists available at ScienceDirect

Mathematical and Computer Modelling

journal homepage: www.elsevier.com/locate/mcm

Analytical mathematical feedback guidance scheme for low-thrust orbital plane change maneuvers

I. Shafieenejad[a,*], A.B. Novinzade[a], R. Shisheie[b]

[a] Department of Aerospace Engineering, K. N. Toosi University of Technology, Tehran, Iran

[b] Mechanical and Aerospace Department, West Virginia University, Morgantown, USA

ARTICLE INFO

Article history:
Received 26 November 2011
Received in revised form 24 June 2013
Accepted 28 June 2013

Keywords:
Optimal control
Analytical closed-loop guidance
Neighboring optimal control
Low-thrust orbital transfer

ABSTRACT

This paper which is based on neighboring optimal control investigates the application of an analytical feedback guidance scheme for the problems of low-thrust orbital transfer. Neighboring optimal control is one of the methods that has been proposed for the closed-loop optimal guidance. This method focuses on expanding the cost function to second order and examining the robustness of the system by the feedback law gains. At first, the open-loop analytical guidance policy is considered as the optimal thrust steering program that will transfer the vehicle from the inclined low earth orbits to the high earth orbits. Secondly, proper feedback optimal guidance laws are analytically obtained to maintain the trajectories around their optimum with assessed disturbances. The proposed guidance scheme is distinguished for two desired performance indices as minimum-time and minimum-effort. Finally performance indices are compared and the best policy is obtained regarding the robustness of the two performance indices against disturbances.

1. Introduction

Optimal control of dynamic systems can be divided into two groups of open-loop and closed-loop optimal controls. Open-loop optimal control is time-dependent and initiates from original conditions, but closed-loop optimal control is state-dependent variables and has robustness against disturbances. In this way, many methods can be illustrated in the field of applied optimal control as dynamic programming, calculus of variations, neighboring optimal control, Pontryagin's min (or max) principle, etc.

Dynamic programming (DP) requires optimal control over an n-dimensional grid near the optimal path. This method needs storing a huge amount of data; by contrast, another method as neighboring optimal control (NOC) needs only storing nominal path and control. Actually, neighboring optimal can be generated as closed-loop optimal control by feed-forward of the control as nominal optimum plus feedback of the deviation. The fundamental of NOC is based on minimizing the second-order terms in the expansion of the performance index function about an optimal nominal trajectory [1].

Based on modification of a well known neighboring optimum feedback control scheme and shooting method, Pesch proposed neighboring optimum guidance of a space shuttle orbiter in Ref. [2]. Next, NOC with respect to a feedback law for the

* Correspondence to: Department of Aerospace Engineering, K. N. Toosi University of Technology, 4th Tehranpars Square, Vafadar St., East, P.O. Box:16765-3381, Postal Code: 16565-83911, Tehran, Iran. Fax: +98 21 77791045.

E-mail addresses: shafiee_iman@yahoo.com (I. Shafieenejad), novinzade@kntu.ac.ir (A.B. Novinzade), reza.shishei@gmail.com (R. Shisheie).

0895-7177/$ – see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.mcm.2013.06.014

Nomenclature

NOC	Neighboring optimal control
$\vec{x}(t)$	States of the system equation
$\vec{u}(t)$	Vector of controls in general
$\dot{\vec{x}}(t)$	Time derivative of states
J	Performance index
t_f	Final time
L	Operator of performance index
$H(t)$	Hamiltonian function
$\lambda(t)$	Co-state
ξ	Magnitude of acceleration
$\beta(t)$	Control variable for low-thrust
$i(t)$	Orbital inclination
$V(t)$	Orbital velocity
$a(t)$	Semi major axes
μ	Gravitational constant
$\delta\vec{u}$	Variation of control vector
$\mathbf{K}(t)$	NOC gain
$\delta\vec{x}$	Variation of state vector
Υ	Division operator for NOC

advanced system was investigated by a robust feedback algorithm for a near minimum-fuel, two stages launch vehicle, by Seywald et al. [3]. A neighboring optimal adaptive method has been proposed by Balakrishnan. He used neighboring optimal control and linear optimal guidance law for his neural network approach [4]. An aero-assist optimal orbital transfer with fuel consumption was devised by a neighboring optimal guidance scheme for a nonlinear dynamic system [5].

Schaub and Alfriend developed an impulsive feedback control to establish specific relative orbits for spacecraft formation flying [6]. In Ref. [7], the technique of neighboring optimal control was proposed to compute near optimal trajectories. In their work, Jardin et al. computed trajectories in general wind fields with nominal solutions based on the Zermelo problem, and the excellent performance of this method was investigated. Also, in Ref. [8] Jardin compared neighboring optimal control to LQR for commercial airlines.

Ref. [9] presented near optimal control laws for minimum-fuel rendezvous between satellites of arbitrary eccentricity in elliptic orbits. Axelrod and Gualman in Ref. [10] designed a feedback law that guided the spacecraft along an optimal neighboring trajectory. Their work was based on electrical propulsion with discrete thrust levels. Gros et al. proposed an approach by considering neighboring extermals for multiple inputs with a feedback law [11].

Hamel and Lafontaine in Ref. [12] developed a feedback control law for neighboring fuel optimal reconfiguration of formation flying spacecraft. Wurth and Hannemann used neighboring extermals to solve dynamic optimization for suitable application in a nonlinear model-predictive control and dynamic real-time optimization [13]. Jiang worked on methods to achieve trajectory with neighboring optimal control for reusable launch vehicle [14]. Liu et al. investigated an optimal control for periodic tasks of discontinuous dynamic systems. They used differential dynamic programming to optimize and generate an optimal control law regarding a linear local model in the neighborhood of the nominal trajectories [15]. In Ref. [16] neighboring extermal regarding to use error compensation is investigated by Kim and Hall.

The optimal low-thrust orbit transfers have received a great amount of attention in flight mechanics and astrodynamics literatures. The evolutions of low-thrust propulsion technologies have reached a point where such systems have become an economical option for many space missions. Also the developments of efficient control laws have received an increasing amount of attention in recent years, and few studies have examined the subject of inclination changing maneuvers [17]. In Ref. [18] the original Edelbaum theory for low-thrusters was revisited within the framework of optimal control for min-time. In Ref. [19], Casalino investigated Edelbaum's approach again, but some improvements were introduced, while maintaining the assumption of quasi-circular orbits. Shafieenejad and Novinzadeh in 2010 had more research works to achieve analytical open-loop solutions for low-thrust orbital maneuvers. They examined and compared different performance indices for low-thrust optimal control problems [20,21].

The current research augments the previous research works [20,21] as studying analytical closed-loop guidance regarding neighboring optimal control. The resulting optimal guidance law is composed of the analytical open-loop controls (based on Shafieenejad's research works [20,21]) and analytical closed-loop time varying guidance law with neighboring optimal control (NOC). Hence, in the middle section of this work, the open-loop optimal low-thrust orbit transfers will be introduced with respect to the previous works and the NOC method will be described. Then, the NOC solutions to the low-thrust as minimum-time and minimum-effort will be investigated. This work concludes many different simulated scenarios to show the robustness of the NOC for two performance indices. Finally, comparisons between these two performance indices will be made with respect to their ability to damp disturbances that are exerted on the system.

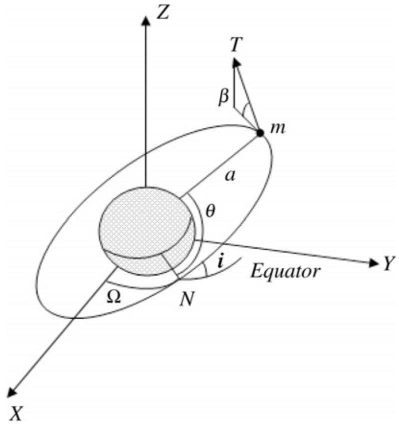

Fig. 1. Low-thrust orbital transfer.

2. Open-loop optimal control for low-thrust orbital transfer

The calculus of variation is utilized in this paper and formulated briefly. At first one can consider the mathematical representations of this system as

$$\dot{\vec{x}}(t) = f\left[\vec{x}(t),\ \vec{u}(t),\ t\right], \quad t_0 \le t \le t_f \tag{1}$$

$\vec{u}(t)$ and $\vec{x}(t)$ are considered as the vector of m-control component and n-state of the system. The conventional form of the cost function is expressed by Eq. (2) without considering the terminal part:

$$J = \int_{t_0}^{t_f} L\left[\vec{x}(t),\ \vec{u}(t),\ t\right] dt. \tag{2}$$

In the calculus of variations, the Hamiltonian of the system is determined as follows:

$$H = L\left(\vec{x}, \vec{u}, t\right) + \lambda^T(t) f\left(\vec{x}, \vec{u}, t\right). \tag{3}$$

$\vec{\lambda}(t)$ demonstrate the co-state of the system. In the next step, the optimality condition of the system is considered as the main principle of optimality $\left(\partial H/\partial \vec{u}\right)^T = 0$. All mentioned equations need to be satisfied simultaneously by the boundary conditions of the optimal control problem. From optimality conditions, m-control components are achieved as functions of $\vec{x}(t)$ and $\vec{\lambda}(t)$. In the most practical applications in optimal control, we confront difficulties of nonlinear equations. In this way, it is very important to facilitate the problem and overcome difficulties of analytical solutions [1,19,20].

3. Low-thrust orbital transfers

In this work, problems of low-thrust orbital transfers are formulated in the framework of optimal control problems. In this formulation the thrust direction is considered as an optimal control variable. The advantages of these guidance laws are that they are the exact solutions of the two-point boundary value problems that can be used later for the analytical closed-loop optimal guidance laws. Also, terminal conditions of time-free plane changing maneuvers of low-thrust spacecrafts are satisfied. Furthermore, in this research, the several difficulties that are associated with the numerical determination of optimal open-loop and closed-loop solutions for the nonlinear systems are eliminated by proposing analytical solutions.

Edelbaum developed simple and applicable equations by using planetary equations. In this way, an analytical optimal control solution is important for space researchers. It should be noted that analytical solutions satisfy boundary conditions and performance measures. There are some suppositions in the Edelbaum equations. The pitch angle is considered as a control variable; orbital inclination and orbital velocity are also considered as state variables. The acceleration of the spacecraft is constant and each orbital revolution is a near-circular orbit regarding the Edelbaum analysis [17].

Minimum-time and minimum-effort performance indices are studied again for non-mass variant, and the results are examined for many maneuvers from LEO to GEO. From the schematic diagram of Fig. 1, $\beta(t)$ is the out-of-plane or thrust angle and ξ represents the magnitude of the acceleration vector. Two components $\xi_t,\ \xi_h$ are considered as $\xi_t = \xi \cos(\beta)$ and $\xi_h = \xi \sin(\beta)$. $\xi_t,\ \xi_h$ represent the tangential and normal components of the acceleration vector respectively. Equations

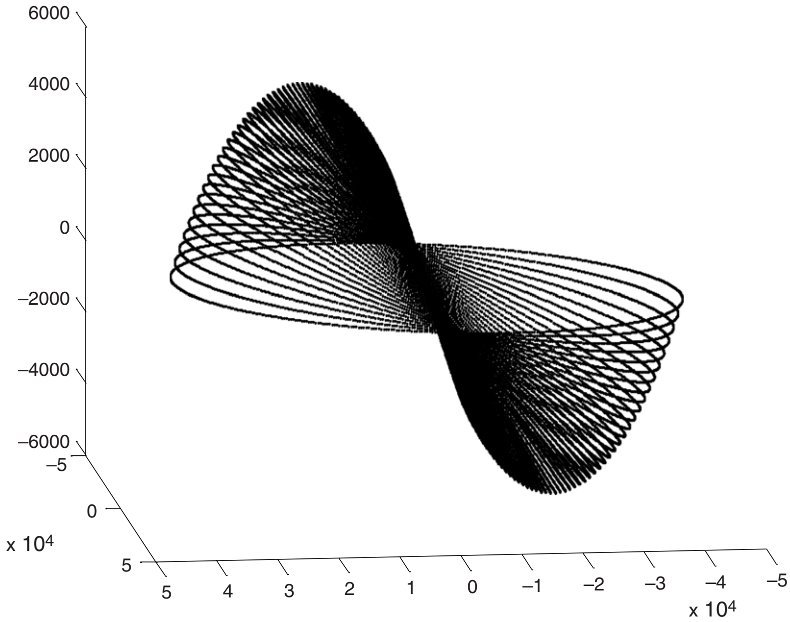

Fig. 2. Optimal orbital maneuver from LEO to GEO.

of this orbital maneuver for low-thrusters are as follows:

$$\begin{cases} \dfrac{di}{dt} = \dfrac{2\,\xi\,\sin(\beta)}{\pi\,V} \\ \dfrac{dV}{dt} = -\xi\,\cos(\beta)\,. \end{cases} \tag{4}$$

The relationship between orbital velocity and semi major axes can be achieved by the energy equation $\frac{V^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}$ [17–20].

3.1. Min-time low-thrust orbital transfers

In this part, the transfer problem is cast as a min-time problem between initial and final conditions i_0, V_0 and i_f, V_f respectively. Co-states λ_i, λ_v are achieved analytically as functions of optimal control respectively: $\lambda_i = \frac{-1}{2}\frac{\pi V\sin(\beta)}{\xi}$, $\lambda_v = \frac{\cos(\beta)}{\xi}$ (see Refs. [20,21]).

The time derivatives appearing in the governing equation can be written with respect to β. In this way, now β becomes the implicit independent variable. $\frac{d\beta}{dt} = \frac{d\lambda_v/dt}{d\lambda_v/d\beta} = \frac{\sin(\beta)\,\xi}{V}$ and the new system equations are given by

$$\begin{cases} \dfrac{di}{d\beta} = \dfrac{2}{\pi} \\ \dfrac{dV}{d\beta} = -\dfrac{V\,\cos(\beta)}{\sin(\beta)}. \end{cases} \tag{5}$$

Due to the system Eq. (5), the results as a function of the control angle β are achieved:

$$V = \frac{\sin(\beta_f)\,V_f}{\sin(\beta)} \tag{6}$$

$$i = i_f + \frac{2\left(\beta - \beta_f\right)}{\pi}. \tag{7}$$

Obviously, it is necessary to specify the values of β_0 and β_f. This can be accomplished through using the known initial and terminal conditions and solving a set of nonlinear algebraic equations. As mentioned, solutions have been expressed in terms of β and it is important to get an analytical closed-loop guidance law based on neighboring optimal control. Fig. 2 shows how optimal orbital maneuver is accomplished around the Earth. For the next step, time-to-go is derived with respect to $\dot{\beta}$

as follows:

$$\text{Time to go} = \frac{V_f \sin(\beta_f - \beta)}{\xi \sin(\beta)}. \tag{8}$$

3.2. Min-effort low-thrust orbital transfers

In this part, after considering minimum-time, the transfer problem is cast as minimum-effort and as in the last section i, V are considered as the state variables. The performance index is considered as $J = \int_{t_0}^{t_f} \beta(t)^2 dt$. Also in this part, co-states are given for this performance index as $\lambda_i = \frac{-1}{2} \frac{\pi V \beta(\sin(\beta)\beta + 2\cos(\beta))}{\xi}$, $\lambda_v = \frac{\beta(\cos(\beta)\beta - 2\sin(\beta))}{\xi}$; see Refs. [20,21].

As mentioned in the previous section, the time derivatives can be derived again with respect to β. On the other hand an implicit relation of control angle and time is obtained; therefore the new system equation is

$$\begin{cases} \dfrac{di}{d\beta} = \dfrac{2\sin(\beta)\left(2+\beta^2\right)}{\pi\beta\left(\sin(\beta)\beta + 2\cos(\beta)\right)} \\ \dfrac{dV}{d\beta} = -\dfrac{V\cos(\beta)\left(2+\beta^2\right)}{\beta\left(\sin(\beta)\beta + 2\cos(\beta)\right)}. \end{cases} \tag{9}$$

If time is considered as an independent variable, exact closed-form solutions that are suitable for neighboring optimal control as analytical feedback guidance law are inaccessible. Hence, state variables of the system equations as functions of control angle help us in the next parts to achieve NOC:

$$V = \frac{V_f \beta_f \left(\sin(\beta_f)\beta_f + 2\cos(\beta_f)\right)}{\beta\left(\sin(\beta)\beta + 2\cos(\beta)\right)} \tag{10}$$

$$i = i_f - \int_{\beta}^{\beta f} \frac{\beta\left(\sin(\beta)\beta + 2\cos(\beta)\right)}{V\left(2+\beta^2\right)} d\beta. \tag{11}$$

Through using the known initial and final conditions and solving a set of nonlinear algebraic equations, β_0 and β_f can be obtained. Time-to-go, according to $\dot{\beta}$, is derived for this performance index:

$$\text{Time to go} = \int_{\beta}^{\beta_f} \left(\frac{V_f \beta_f \left(2+\beta^2\right)\left(\sin(\beta)\beta + 2\cos(\beta)\right)}{\beta^2 \xi \left(-4\beta\sin(\beta)\cos(\beta) - 4\cos(\beta)^2 - \beta^2 + \beta^2\cos(\beta)^2\right)} \right) d\beta. \tag{12}$$

4. Closed-loop optimal control via the neighboring optimal control (NOC) method

One of the main problems that plays an important role in space-systems' motion is determining the optimal trajectory and providing a proper closed-loop optimal guidance law to maintain the system while disturbances are being assessed. For the first step to achieve this purpose, the solution of the desired problem based on the open-loop optimal control theory should be determined. In this way, the calculus of variation is one of the most important methods to provide proper solutions for optimal control problems. This method takes even more credit when the problem has a nonlinear behavior as the presented problem in this research. Providing an analytical optimal solution is very difficult due to complexities of equations and advanced mathematical methods; however, achieving an analytical optimal solution for the desired problem is very useful and it can assist us to achieve analytical closed-loop and enhance the robustness of the system against disturbances.

The purpose of this part is providing a modified optimal control vector to damp the exerted disturbances based on an analytical neighboring closed-loop control method. This method which results in the time-variant feedback control law minimizes the second-order optimality criterion of the nominal trajectory disturbances. According to the NOC method, the gain feedback can be planed as a time-interval function between the end time and the current time, which predicts the variation of final time. One of the advantages of this method, unlike other methods such as *LQR*, is the possibility to modify the end time. It should be noted that the *NOC* method is based on a feedback algorithm which means that the control variable can be reproduced and rectified by $\delta\vec{u}$ [1,7]:

$$\delta\vec{u}(t) = \mathbf{K}_u(t)\delta\vec{x}(t) \tag{13}$$

where $\delta\vec{u}(t)$ represents the variation of the control vector which generated by the gains and $\delta\vec{x}$ denotes the disturbed state vector. Also $\mathbf{K}(t)$ is the neighboring time-variant optimal feedback gain.

Since the end time of the problem in calculations is not specified, the technique presented in this paper can be employed to calculate the end-time variations based on the gains and rectify the nominal optimal response subsequently:

$$dt_f = \mathbf{K}_t(t)\delta\vec{x}(t). \tag{14}$$

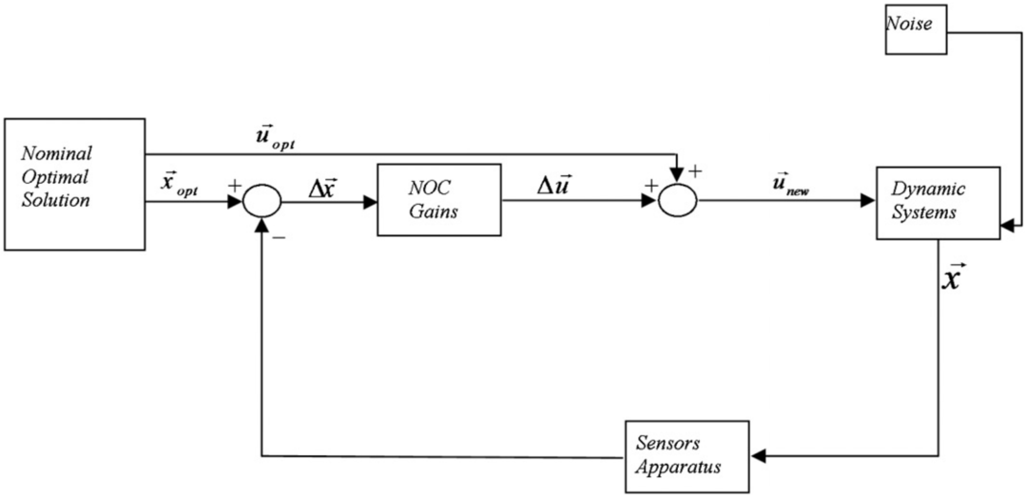

Fig. 3. Schematic plan of the closed-loop law based on the NOC.

According to this method, the feedback rule for β and β_f as control variable and the value of the control at end time respectively are expressed as the below matrix equation:

$$\begin{pmatrix} \delta\beta \\ \delta\beta_f \end{pmatrix} = \begin{pmatrix} \dfrac{\partial\beta}{\partial x_1} & \dfrac{\partial\beta}{\partial x_2} \\ \dfrac{\partial\beta_f}{\partial x_1} & \dfrac{\partial\beta_f}{\partial x_2} \end{pmatrix} \begin{pmatrix} \delta x_1 \\ \delta x_2 \end{pmatrix}. \tag{15}$$

So the disturbances exerted to variables such as $\vec{X}$ are defined as follows:

$$\delta\left(\vec{X}\right) \equiv \left(\vec{X} - \vec{X}_{\text{nomi}}\right). \tag{16}$$

On the other hand, the partial differential of β and β_f to state variables are

$$\frac{\partial\beta}{\partial x_1} = \left(\frac{1}{\Upsilon}\right)\frac{\partial x_2}{\partial\beta_f} \tag{17}$$

$$\frac{\partial\beta}{\partial x_2} = \left(\frac{-1}{\Upsilon}\right)\frac{\partial x_1}{\partial\beta_f} \tag{18}$$

$$\frac{\partial\beta_f}{\partial x_1} = \left(\frac{-1}{\Upsilon}\right)\frac{\partial x_2}{\partial\beta} \tag{19}$$

$$\frac{\partial\beta_f}{\partial x_2} = \left(\frac{1}{\Upsilon}\right)\frac{\partial x_1}{\partial\beta}. \tag{20}$$

The presented division operator Υ in the above equations is also defined as follows:

$$\Upsilon = \left(\frac{\partial x_1}{\partial\beta}\right)\left(\frac{\partial x_2}{\partial\beta_f}\right) - \left(\frac{\partial x_1}{\partial\beta_f}\right)\left(\frac{\partial x_2}{\partial\beta}\right). \tag{21}$$

Finally, control variables with regard to disturbances are defined as (see Fig. 3)

$$\vec{u}_{\text{new}} = \vec{u}_{\text{opt}} + \delta\vec{u}. \tag{22}$$

And in the next step, final time variation according to the following equation, where T is time as a function of β, β_f, is demonstrated [1,7]:

$$dt_f = \left[\left(\frac{\partial T}{\partial\beta}\frac{\partial\beta}{\partial x_1} + \frac{\partial T}{\partial\beta_f}\frac{\partial\beta_f}{\partial x_1}\right)\delta x_1 + \left(\frac{\partial T}{\partial\beta}\frac{\partial\beta}{\partial x_2} + \frac{\partial T}{\partial\beta_f}\frac{\partial\beta_f}{\partial x_2}\right)\delta x_2\right]_{t=t_f}. \tag{23}$$

Calculating the above equation necessitates achieving final time and states with respect to β, β_f.

In the present paper an analytical method is employed to get feedback gain; however, other methods like backward sweep integration that is presented in Ref. [1] confront researchers with computational problems. But due to analytical

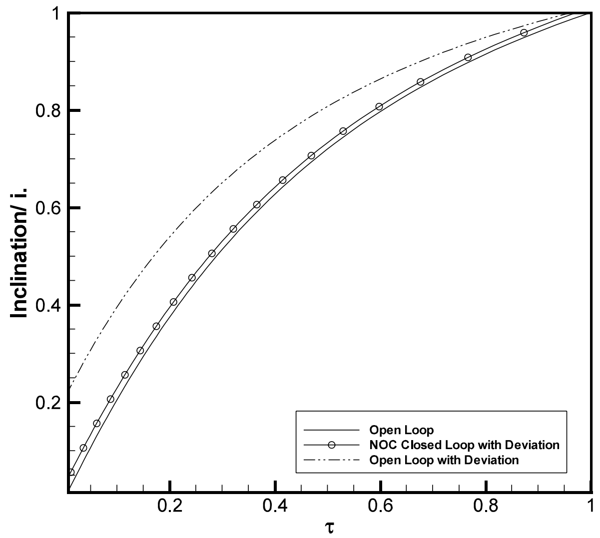

Fig. 4. NOC response of the orbital inclination based on disturbance of type (I) for min-time.

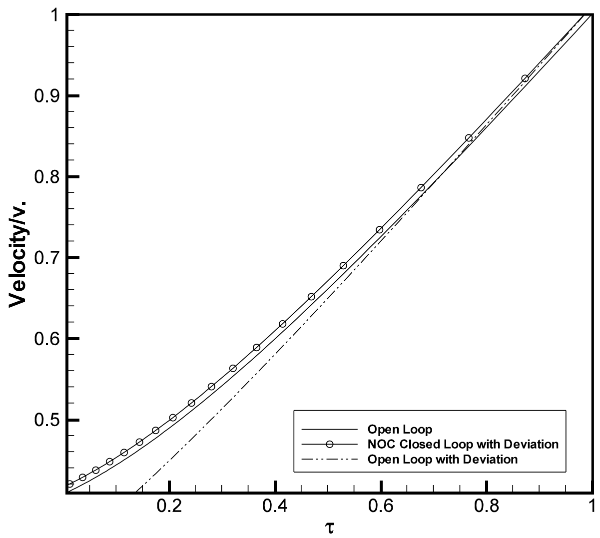

Fig. 5. NOC response of the orbital velocity based on disturbance of type (I) for min-time.

solutions provided for this problem, differentiation regarding no-singularity-existence along the time domain is easier and more suitable. It should be noted that this algorithm is applicable for problems that the end time is not fixed.

According to the method proposed by the optimal neighboring method, the desired gains will be achieved and we can control the space system about the near optimal trajectory by multiplying those gains by the deviation of the state variables:

$$\delta u = K_1(t)\,\delta x_1 + K_2(t)\,\delta x_2. \tag{24}$$

As shown in Fig. 3, the new state variables are obtained through applying disturbances to a system and measuring the resulted values. Subsequently the new gain which leads us to compute the offset values of the control variable or $\delta\vec{u}$ is achieved by comparing the new values with the optimum values of state variables. The summation of this value and the basic $\vec{u}_{\text{opt}}$ value which is obtained through the open-loop optimal control, and considering the deviation of state variables from the open-loop optimal, results in a new control variable $\vec{u}_{\text{new}}$.

4.1. Closed-loop optimal control of a low-thrust spacecraft orbit transfer with min-time criterion

As described before, open-loop optimal control results based on min-time criterion as a function of β and β_f are considered primarily (β is the control variable). Considering Eq. (15), $\delta\beta$ and $\delta\beta_f$ will be achieved by applying the Υ operator

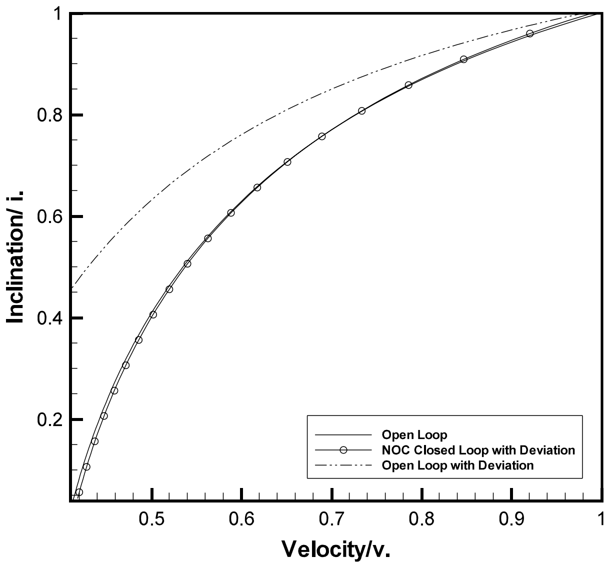

Fig. 6. NOC response of the orbital velocity and orbital inclination based on disturbance of type (I) for min-time.

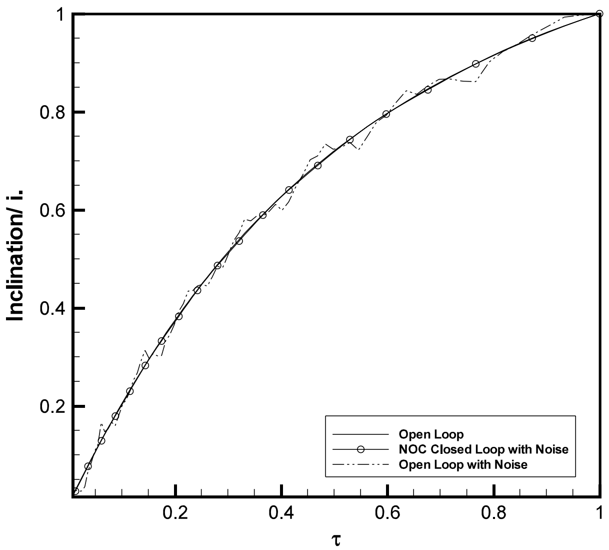

Fig. 7. NOC response of the orbital inclination based on disturbance of type (II) for min-time.

as defined in Eqs. (17)–(21). In the next step, the value of dt_f based on Eq. (23) is achieved and the new control variable $\delta\beta(t) = \beta_{\text{nomi}} + \delta\beta$ is resulted according to the defined algorithm (Fig. 3).

The equations that are presented in this paper for analytical closed-loop clearly indicate that the end time is not fixed. Also, it is necessary to rectify the values of final time t_f and the final angle β_f that are presented in Table 1. It should be noted that calculating new β_f, t_f is one of the advantages of the NOC method against other methods to get feedback gains:

$$a_0 = 7000\,(\text{km}) \qquad a_f = 42{,}000\,(\text{km}) \qquad i_0 = 28.5^\circ \qquad i_f = 0^\circ \qquad \xi = 3.5 \times 10^{-7}\left(\frac{\text{km}}{\text{s}^2}\right).$$

With respect to Ref. [16], to verify the closed-loop performance based on the neighboring method, it is proper to apply deviations and disturbances to the system and try to guide the system about the near optimal neighborhood. Therefore, in this part, two types of disturbances, as initial deviations and noises, are applied to the system in order to study the robustness of the system. Disturbances of type (I) are modeled as initial deviation with amplitude of 0.01 values of states as $\delta i_0 = [0.01]\, i_0$, $\delta V_0 = [0.01]\, V_0$. In type (II) a sinusoidal disturbance which has an amplitude of 0.05 of control variable is exerted on the control variable to investigate the effect of NOC gains. State variables are achieved in the next figures for three different manners: the open-loop optimal solution without disturbance, the open-loop optimal with disturbance and neighboring optimal control solutions (NOC).

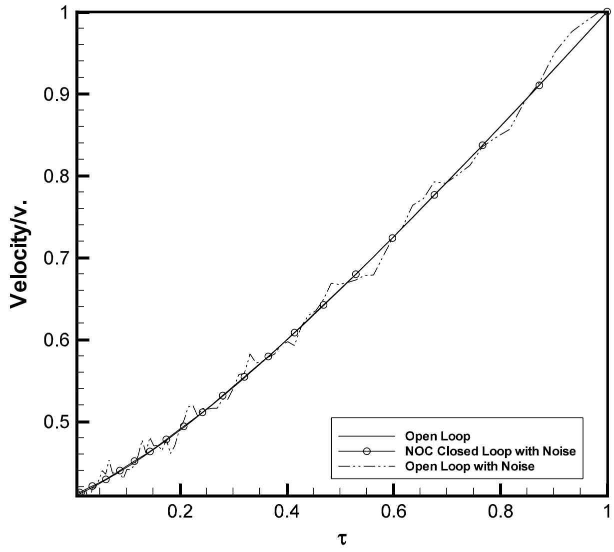

Fig. 8. NOC response of the orbital velocity based on disturbance of type (II) for min-time.

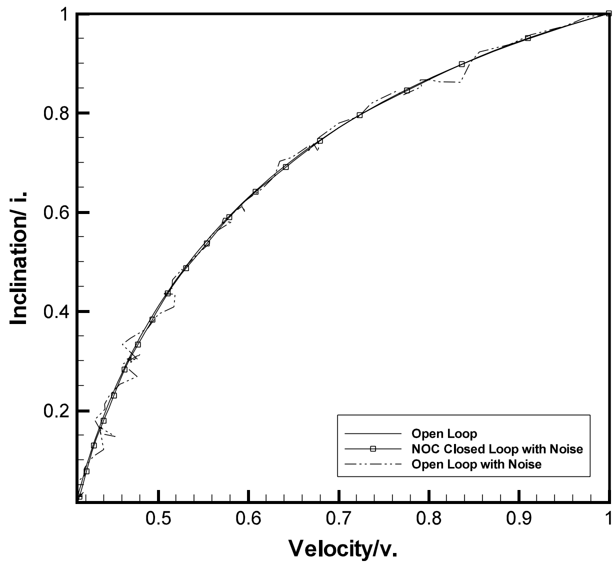

Fig. 9. NOC response of the orbital velocity and orbital inclination based on disturbance of type (II) for min-time.

Table 1
Open-loop and closed-loop optimal control results based on the NOC method.

Solutions	Min-time
Open-loop optimal control	
Open-loop final time (day)	191.2738
β_0 (°)	21.9911
β_f (°)	66.7838
$\Delta\beta$ control command (°)	44.7927
Closed-loop feedback guidance law	
NOC final time (day) (disturbance type I)	191.2759
NOC β_f (°) (disturbance type I)	66.7927
NOC final time (day) (disturbance type II)	191.2784
NOC β_f (°) (disturbance type II)	66.7935

Table 2
Open-loop and closed-loop optimal control results based on the NOC method for min-effort.

Solutions	Min-control-effort
Open-loop optimal control	
Open-loop final time (day)	191.3755
β_0 (°)	23.8226
β_f (°)	61.5515
$\Delta\beta$ control command (°)	37.72887
Closed-loop feedback guidance law (NOC)	
NOC final time (day) (disturbance type I)	191.3769
NOC β_f (°) (disturbance type I)	61.6194
NOC final time (day) (disturbance type II)	191.4078
NOC β_f (°) (disturbance type II)	61.6236

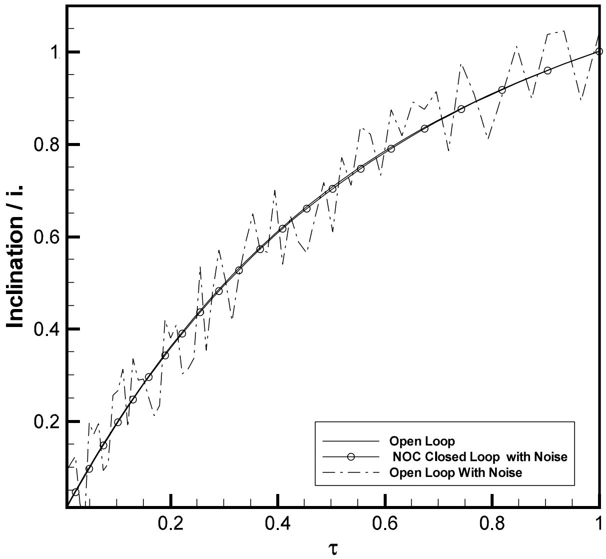

Fig. 10. NOC response of the orbital inclination based on disturbance of type (I) for min-effort.

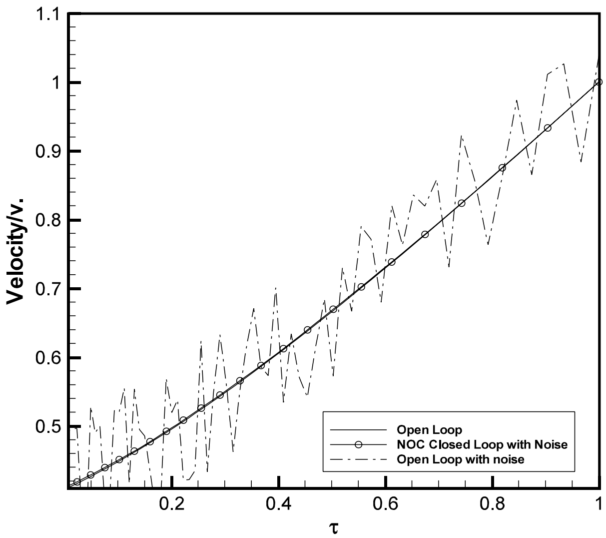

Fig. 11. NOC response of the orbital velocity based on disturbance of type (I) for min-effort.

As shown in Figs. 4–9, the presented guidance law can guide the spacecraft about the neighbored of the open-loop optimal trajectory.

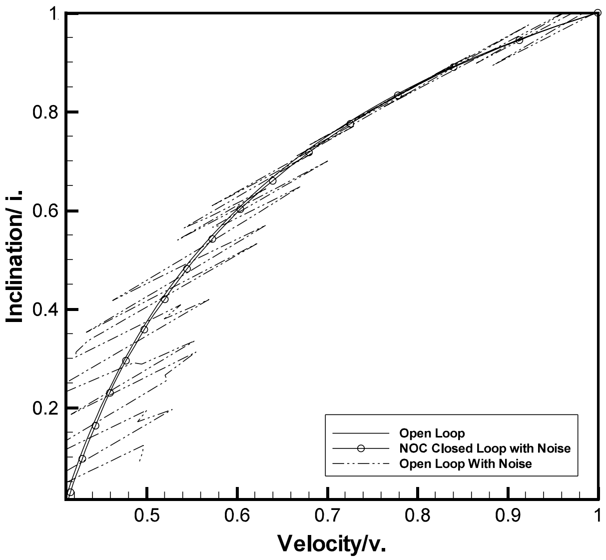

Fig. 12. NOC response of the orbital velocity and orbital inclination based on disturbance of type (I) for min-effort.

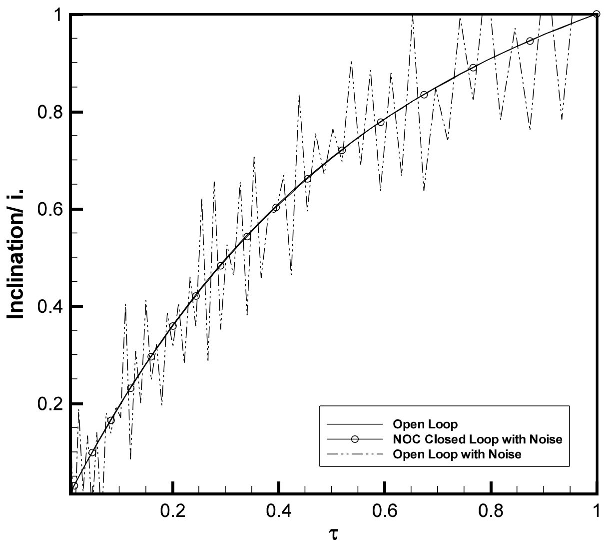

Fig. 13. NOC response of the orbital inclination based on disturbance of type (II) for min-effort.

4.2. Closed-loop optimal control of low-thrust orbital transfer with the min-effort criterion

In this section, the analytical closed-loop of the orbital transfer problem based on a min-effort criterion is investigated. As the first step according to the open-loop optimal solutions of the previous part (see Eqs. (10), (11)), the state variables as a function of β and β_f are rewritten. Then the feedback gains are achieved by considering the NOC closed-loop Eqs. (15)–(21). As shown in Fig. 3, disturbance is exerting on the system and its effect on the state variable is considered. Also Figs. 10–15 demonstrate that the open-loop solution has high sensitivity to disturbance and a satellite tends to deviate from the optimal trajectory, but in the closed-loop control solution, all disturbances have been damped perfectly. Hence, the optimal trajectory is very close to the optimal solution without disturbances. To study the effect of disturbance on the state variables, the following disturbances in two types are added to the system. Disturbance of type (I) is modeled as a sinusoidal disturbance with amplitude value of 0.1. In type (II), a random disturbance which has a normal distribution, unit variance and zero mean value is exerted. Finally, boundary conditions are the same as what are stated in Table 2 and also, the variation of β and β_f regarding the applied disturbances to the state variables are taken into consideration:

$$a_0 = 7000\,(\text{km}) \qquad a_f = 42{,}000\,(\text{km}) \qquad i_0 = 28.5^\circ \qquad i_f = 0^\circ \qquad \xi = 3.5 \times 10^{-7}\left(\frac{\text{km}}{\text{s}^2}\right).$$

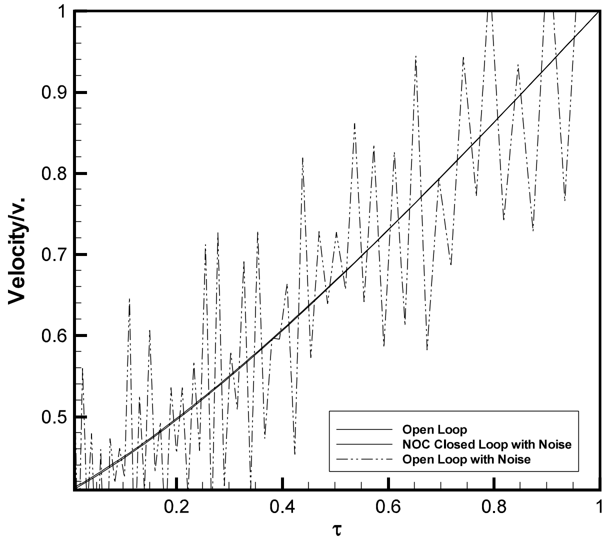

Fig. 14. NOC response of the orbital velocity based on disturbance of type (II) for min-effort.

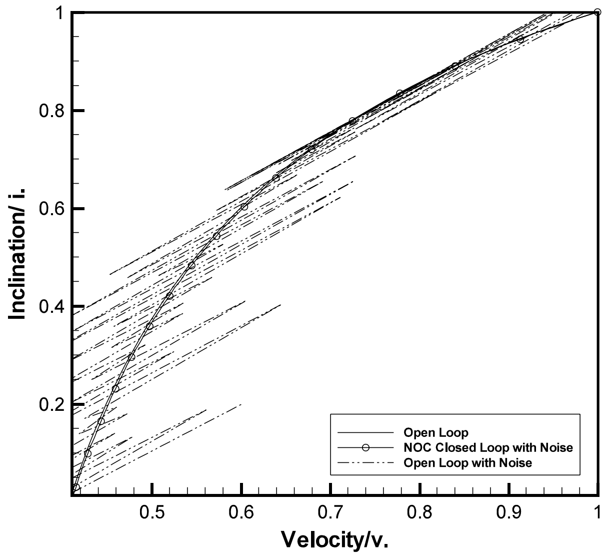

Fig. 15. NOC response of the orbital velocity and orbital inclination based on disturbance of type (II) for min-effort.

5. Conclusions

In this research, optimal control problems for two performance indices as min-time and min-effort are introduced. Secondly these two indices for low-thrust orbital maneuvers are investigated to achieve analytical closed-loop guidance laws by Neighboring Optimal Control (NOC). This method needs only storing nominal path and optimal control. By feed-forward of the control as nominal optimum plus feedback of the deviation, neighboring optimal can be generated as closed-loop optimal control. It is concluded from Tables 1, 2 that the min-effort criterion is more appropriate than the min-time criterion regarding $\Delta\beta$ and final time. The results of this work demonstrate the fact that the closed-loop optimal response has a better effect on the min-effort and it can damp the disturbance with higher amplitude for this criterion. In other words, it is one of the novelties of this work to assign a better performance index than min-time regarding the open-loop and closed-loop solutions for low-thrust spacecraft maneuvers. As shown in Tables 1, 2, t_f increases by exerting disturbances on the system and it can be computed in this method. Also, variations of control value at the end of the trajectory are determined. Finally, the results illustrate the viability of the NOC method as analytical closed-loop guidance laws for low-thrust orbital maneuvers regarding the robustness of these guidance methods against disturbances.

References

[1] A.E. Bryson, Dynamic Optimization, Addison-Wesley Longman Inc., California, 1999.
[2] H.J. Pesch, Neighboring optimum guidance of a space-shuttle-orbiter-type vehicle, J. Guid. Control Dyn. 3 (1980) 386–391.
[3] H. Seywald, E.M. Cliff, Neighboring optimal control based feedback law for the advanced launch system, J. Guid. Control Dyn. 17 (1994) 1154–1162.
[4] J. Dalton, S.N. Balakrishnan, A neighboring optimal adaptive critic for missile guidance, J. Math. Comput. Modelling 23 (1996) 175–188.
[5] D.S. Naidu, J.L. Hibey, C.D. Charalambous, Neighboring optimal guidance for aero assisted orbital transfer, IEEE Trans. Aerosp. Electron. Syst. 29 (1993) 656–665. http://dx.doi.org/10.1109/7.220918.
[6] H. Schaub, K.T. Alfriend, Impulsive feedback control to establish specific mean orbit elements of spacecraft formation, J. Guid. Control Dyn. 24 (2001) 739–745.
[7] M. Jardin, A.E. Bryson, Neighboring optimal aircraft guidance in winds, J. Guid. Control Dyn. 24 (2001) 710–715.
[8] M. Jardin, Ideal free flight through multiple aircraft neighboring optimal control, in: Proc. of the American. Control. Conf., American. Automatic. Control., 2000, pp. 2879–2885. http://dx.doi.org/10.1109/ACC.2000.878737.
[9] R. Sharma, P. Sengupta, S.R. Vadali, Near-optimal feedback rendezvous in elliptic orbits accounting for nonlinear differential gravity, J. Guid. Control Dyn. 30 (2007) 1803–1813.
[10] A. Axelrod, M. Gualman, Optimal control of interplanetary trajectories using electrical propulsion with discrete thrust level, J. Guid. Control Dyn. 25 (2002) 932–939.
[11] S. Gros, B. Srinivasan, D. Bonvin, Robust predictive control based on neighboring extremals, J. Process Control 16 (2006) 243–253.
[12] J.F. Hamel, J. Lafontaine, Neighboring optimum feedback control law for earth-orbiting formation-flying spacecraft, J. Guid. Control Dyn. 32 (2009) 290–299.
[13] L. Würth, R. Hannemann, W. Marquardt, Neighboring-extremal updates for nonlinear model-predictive control and dynamic real-time optimization, J. Process Control 19 (2009) 1277–1288.
[14] Z. Jiang, R. Ordoez, Extension of motion primitives and neighboring optimal control used in trajectory generation for RLVs, in: Proc. IEEE 2009 Aero & Elect. Conf. (NAECON), 2009, pp. 219–222. http://dx.doi.org/10.1109/NAECON.2009.5426623.
[15] C. Liu, C.G. Atkenson, J. Su, Neighboring optimal control for periodic tasks for systems with discontinuous dynamics, J. Sci. China 53 (2010) 1–18.
[16] T.J. Kim, D.G. Hull, Neighboring extremal control design using error compensation, J. Guid. Control Dyn. 20 (1997) 542–548.
[17] J.C. Bastante, A. Caramagno, Low thrust transfer optimization of satellites formations to heliocentric earth trailing orbits through a gradient restoration algorithm, J. Acta. Astronom. 55 (2004) 497–507.
[18] J.A. Kechichian, Reformulation of Edelbaum's low-thrust transfer problem using optimal control theory, J. Guid. Control Dyn. 20 (1997) 988–994.
[19] L. Casalino, G. Colasurdo, Improved Edelbaum's approach to optimize low earth/geostationary orbits low-thrust transfers, J. Guid. Control Dyn. 30 (2007) 1504–1511.
[20] I. Shafieenejad, A.B. Novinzadeh, Analytical solutions for two-point boundary value problems: optimal low-thrust orbit transfers, Proc. Inst. Mech. Eng. G 224 (2010) 843–853.
[21] I. Shafieenejad, A.B. Novinzadeh, Closed form optimal continuous guidance scheme for new dynamic of electrical propulsion plane change transfers, Proc. Inst. Mech. Eng. K 224 (2010) 233–241.